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APPLICATION OF MIXED STRATEGIES FOR IMPROVED MISSILE GUIDANCE - VALIDATION FOR VARIABLE SPEED MODELS

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This technical report has been reviewed and is approved for publication.

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SUMMARY

This report presents the results of the third year research efforts of the Grant. The investigations of the two previous years dealt with a constant velocity missile model. The last year activities were oriented to evaluate the validity of the previously obtained results for more realistic variable speed missile models. For this purpose an aerodynamic model provided by WL/MNAG was used. Moreover, acceleration limits were imposed separately on each guidance channel, simulating trimmed angle of attack limits. The study indicated that the guidance law used in the previous (constant speed) analysis has to be modified and incorporate a term for compensating the effect of missile longitudinal acceleration. With this modification the homing performance of a variable speed missile using mixed strategy guidance is very similar to the performance of a constant speed model, as long as the terminal maneuverability of the missile remains unchanged.

I. INTRODUCTION

During the first year of the three year grant [1] a Mixed Strategy Guidance Law (MSG) was developed for a three-dimensional end-game scenario with eventual electronic countermeasures (ECM). The operational effectiveness of MSG was compared with conventional missile guidance laws, such as Proportional Navigation (PN) and Augmented Proportional Navigation (APN). The comparison clearly demonstrated that MSG had a much superior effectiveness than the other guidance laws. The objective of the second year research activity was to generalize these results, obtained for a single engagement geometry (head-on) and for a single set of missile and aircraft parameters, as well as to extend the validity of the comparison by a parametric investigation based on a non-dimensional sensitivity analysis. The sensitivity analysis [2] lead to conclude that, if the mixed strategy guidance (MSG) approach is used, a missile-target maneuver ratio of 3 is sufficient for a robust operational effectiveness. The effects of engagement geometry (initial target aspect angle and range) on the homing performance of the "pure guidance strategies", which constitute the basis of MSG, were similar to the well known results predicted by the classical linearized guidance theory.

All these previous results were based on a constant speed missile model, a frequently used assumption in many investigations and one of the elements in linearized guidance analysis. Unfortunately, for the terminal guidance phase of an air-to-air missile this assumption is not valid. Long and medium-range guided missiles reach their targets in an unpropulsed mode, loosing kinetic energy (i.e. speed) for overcoming aerodynamic drag. Depending on speed and altitude the missile deceleration can reach the level of 2-5 g's. Moreover, the maneuverability of the missile is also a function of the dynamic pressure. For these reasons it seemed very important to evaluate the validity of the results obtained for a constant speed missile model in the more realistic variable speed

scenario. For this purpose an aerodynamic (lift and drag) model of the missile was incorporated in the end-game simulation. (This model was provided to us by WL/MNAG.) Assuming a skid-to-turn aerodynamic configuration, the lateral acceleration limit, imposed in the previous phases on the resultant acceleration command, was replaced by limits imposed on each guidance channel separately.

The assessment of the variable speed model was performed in two phases. In the first phase the initial speed was selected in order to allow the final dynamic pressure to be large enough for achieving the maximum acceleration command. In the next phase a lower initial speed was selected leading to a reduced final maneuverability.

For sake of completeness, the next section repeats the problem formulation and gives a brief summary of previous results. In section III the main results of the investigation are summarized including a performance comparison at each step.

II. BACKGROUND INFORMATION

2.1 PROBLEM FORMULATION

The investigation deals with the terminal phase of an encounter between a radar guided missile and a maneuvering aircraft which has the capability to use electronic countermeasures (ECM). This problem has been formulated as a two-person, zero-sum, imperfect information game in which both players may use mixed strategies, [3].

In the investigation the following terminology is used:

a. The missile and the target aircraft called "pursuer" and "evader", respectively.

b. A "guidance law" is the mapping of the estimated state into acceleration commands.

c. The tandem combination of an estimator and a guidance law is called a "pure

guidance strategy".

d. A pure "evader strategy" is a combination of a feasible maneuver sequence and some countermeasure policy.

e. A "mixed strategy" is a probability distribution over a set of pure strategies.

Our investigation addresses this problem from the point of view of the missile designer who wishes to determine, for a given set of evader strategies, the optimal pure strategy set and the corresponding mixed strategy of the pursuer that maximizes the minimal value of the performance measure, which is the "Single Shot Kill Probability" (SSKP) of the missile.

The result of this maximum operation over the set of possible outcomes associated with given pure strategies is V_m (the value of the mixed strategy game), which can be interpreted as the "guaranteed SSKP" of the missile, as well as the corresponding probability distributions (optimal mixed strategies).

Scenario Description

The main elements of the assumed scenario are:

1) The encounter is three-dimensional (see Fig. 1).

2) The game starts and takes place in the vicinity of the collision course and terminates when the range rate becomes zero.

3) In order to enhance survivability, the evader may make use of electronic contrameasures (ECM) simultaneously with its maneuver. The ECM technique considered in this work is electronic jinking (EJ) [4]. It generates a deterministic motion of the aircraft's radar deflection center from wing-tip to wing-tip. Whenever EJ is applied, the inherent stochastical fluctuation of the radar reflection center, called the "glint" noise, becomes hardly observable.

4) For the sake of simplicity, it is assumed that the motion of the evader is

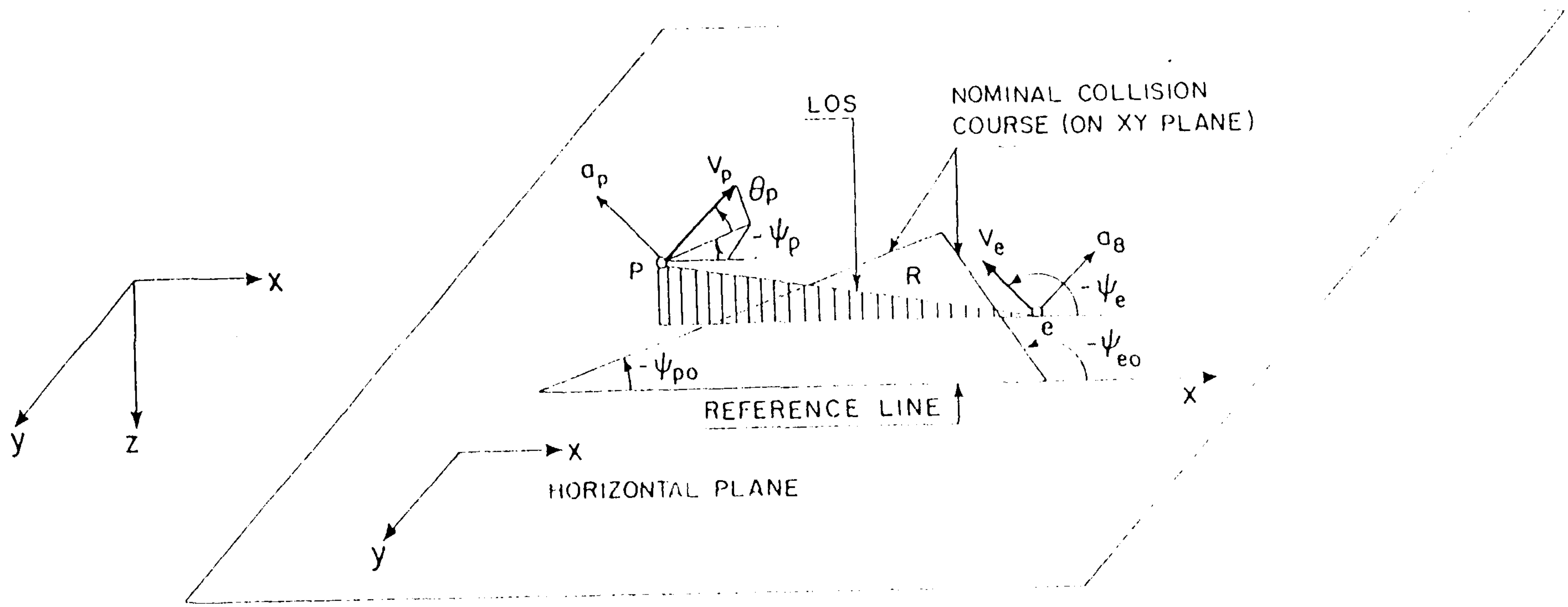

Figure 1. Engagement geometry.

confined to the collision plane (assumed to be horizontal). Note that even if this assumption is adopted and effects of gravity are neglected, the motion of the pursuer will be three-dimensional because the disturbances caused by EJ or the "glint" noise have a predominant transversal component [5].

Information Structure

Throughout the duration of the game the pursuer measures the relative range R, the closing velocity V_c, and the line of sight angle λ, relative to a reference line. It is assumed that the range and velocity measurements are exact and that the angle measurement is corrupted by noise. The velocity and range information is processed to give an accurate estimate of the "time-to-go", while the range and angle information is processed and yields a "noisy measurement" of the evader's relative position perpendicular to the reference line. The angular measurement is perturbed either by the "glint" noise, or by EJ intentionally generated by the evader.

The evader knows when the game starts, but has neither measurements on the state of the game nor knowledge of the duration of the game.

The rules of the game are such that at the beginning of the game (or shortly before it), each player "selects" through a chance mechanism one of its pure strategies and plays according to it until the end of the game.

2.2 MODELING

Evader Model

It is a constant speed point-mass model with 3 options of lateral acceleration: (i) No maneuver, (ii) Constant maneuver, (iii) Random phase periodical maneuver.

The roll dynamics of the evader is taken into account by the time t_R required to change the direction of the maneuver.

The electronic jinking (EJ) is also periodical with random phase,

uncorrelated with the kinematical maneuver.

Pursuer Model

The pursuer is a radar guided homing missile represented by a simplified mathematical model. The kinematics of the missile is of a point-mass model with limited lateral acceleration. Moreover, it is assumed that the three-dimensional interception is accomplished by two identical decoupled guidance channels operating in perpendicular planes. Each guidance channel model consists of the following elements:

1) The "seeker", which reconstructs the line of sight direction from radar measurements corrupted by "glint" type noise and provides the relative lateral displacement with respect to a stabilized reference. The generation of this signal doesn't involve any delay.

2) The "estimator" extracts from the noisy signal of the "seeker" a smoothed estimate of the relative state.

3) The "guidance computer" determines the required lateral acceleration, the command signal of the "autopilot", taking into account the acceleration limit.

4) The "autopilot" and lateral missile dynamics are approximated (as a closed-loop system) by a first-order transfer function between the required and actual lateral acceleration.

Lethality Model

The probability of destroying the target (P_K) is a single valued function of the actual miss distance determined by two parameters: the overall reliability of the guidance system $(P_K)_{max}$ and a characteristic lethal radius R_ℓ. If the miss distance R_f is smaller than R_ℓ then $P_K=(P_K)_{max}$. For $R_f>R_\ell$ the following functional relationship is used.

$$P_K(R_f) = (P_K)_{max} \exp\left\{- 4 \left(\frac{R_f}{R_\ell} - 1\right)^2 \right\} .$$

In an imperfect information scenario the miss distance is a random variable. For a given pair of pure strategies, $(\delta_e)_i$ and $(\delta_p)_j$, the "Single Shot Kill Probability" (SSKP) is expressed by

$$P_{ij} = E\left\{P_K(R_f) \mid (\delta_e)_i \; (\delta_p)_j\right\} \quad ,$$

where the expectation is taken over the ensemble of all measurement noise samples, as well as the random phases of the target maneuver and the EJ.

2.3 MIXED STRATEGY GUIDANCE CONCEPT

Application of the Mixed Strategy Guidance (MSG) approach generates an optimal mixed strategy based on an optimal set of pure guidance strategies. Each of the pure guidance strategies is composed of two elements: a "perfect information guidance law", developed on the basis of a linearized differential game model [6], and an "estimator". The input for this guidance law is the zero-effort miss distance, is based on the line of sight rate and a compensation of own lateral acceleration. It is independent of the evader acceleration as implied by differential game theory. Since the scenario is noise corrupted, the actual input of the guidance law comes from the "estimator", in the form a steady-state Kalman filter. Though assumptions on the evader maneuver's are incorporated in the dynamic model of the "estimator", the estimated zero-effort miss distance doesn't include the evader acceleration. For a given structure, the "estimator" is determined by a set of parameters to be selected by the designer. The maximum dimension of such an "estimator" is either 4 or 6 depending on whether EJ is considered or not (see Fig. 5 in Ref. 1). Each "estimator" design combined with the "perfect information guidance law" forms a "pure guidance strategy".

In the "estimator" the random target maneuver and the eventual "electronic jinking" are considered as stochastic processes, each approximated by a "shaping filter" fed by white noise [7]. In the present investigation, each "shaping

filter" has the following form

$$G_i = \frac{s+\alpha_i}{s^2+2\zeta_i\tilde{\omega}_i s+\tilde{\omega}_i^2} \quad ; \qquad i=e, j$$

where the subscript "e" stands for evader maneuver and "j" for electronic jinking. Each shaping filter is fed by white noise of a given spectral density Φ_i (i=e, j) where

$$\Phi_e = k_e(g\ n_{max})^2$$

and

$$\Phi_j = k_j(w_{max})^2$$

In addition to the two sets of four parameters (k_i, α_i, ζ_i, $\tilde{\omega}_i$) one has to consider also Φ_{gl} the spectral density of the "glint", which is the inherent measurement noise. Thus the designer has (in the limitations imposed by the structure of the filter) a set of 9 parameters, which in a way constitute the assumption made on the behavior of the evader. Based on these parameters, the gains of the Kalman filter are computed by solving the appropriate algebraic Riccati equation.

2.4 SUMMARY OF PREVIOUS RESULTS

First year

The investigation performed in the first year was reported in Ref. 1. For a combined ECM/no-ECM scenario against a set of 90 pure evader strategies and for a given set of engagement parameters, a Mixed Strategy Guidance (MSG) law called M*, based on a pair of "pure guidance strategies", designated respectively "J" and "L", was found to be satisfactory. The parameters of the shaping filters for these guidance strategies were determined as follows:

Strategy "J": Φ_{gl}=0.04

k_e =6.3	α_e =1.8 $\frac{1}{sec}$	ζ_e=0.151	$\tilde{\omega}_e$=0.78 r/sec
k_j =0.35	α_j =38.0 $\frac{1}{sec}$	ζ_j=0.155	$\tilde{\omega}_j$=0.70 r/sec

Strategy "L": $\Phi_{gl}=1.0$ $k_j=0$

$k_e=0.10$ $\alpha_e = 0.5 \frac{1}{sec}$ $\zeta_e=0.15$ $\tilde{\omega}_e=0.1$ r/sec

In this environment and for the given set of parameters MSG achieved a guaranteed SSKP of slightly higher than 0.4, ($V_m=0.402$) compared to much lower values (below 0.1) achieved by PN and APN.

The PN guidance law in the comparison had a constant "effective proportional navigation gain" N'=3.0 and the estimated line of sight rate was obtained from the seeker via a first-order low-pass filter with a time constant of 0.3 sec.

In the APN, which includes the estimate of the evader's lateral acceleration as a component of the estimated state, the guidance gain of N'=3 was selected. The estimator was a steady-state Kalman filter based on a "random telegraph" type (first-order) target maneuver model with the parameter $\lambda_t=0.5$ 1/sec.

Second year

The research effort of the second year is summarized in Ref. 2. It was oriented to test the sensitivity of the mixed strategy guidance (MSG) concept to several parameters of interest. In order to generalize the validity of the analysis the results are presented in a nondimensional form as functions of the similarity parameters of the problem. The investigation concentrated on evaluating the sensitivity of MSG to the engagement geometry, pursuer-evader maneuver ratio and warhead lethality. The strategy set of the evader, as well as the relationships between the different "disturbance" element such as glint noise, target maneuverability, electronic jinking were kept constant by preserving the values of the respective similarity parameters. Therefore no change in the "optimal estimation" has been expected in the sensitivity analysis. Since against the given evader strategy set the best estimator pair

was found in the first year investigation [1], any new optimal mixed strategy in the same environment was based on the same pure strategy pair (namely "J" and "L").

First the guidance performance (miss distance) sensitivity of the two pure guidance strategies "J" and "L" (composing the optimal strategy pair for the MSG) to variations of the initial conditions of the engagement, such as initial target aspect angle and initial range (or time of flight), was tested.

Both sensitivity tests lead to results which agree well with the phenomena encountered in linearized guidance theory. It could be thus concluded that in a rather large domain of initial conditions the concept of MSG and its performance is not sensitive to the engagement geometry.

The sensitivity analysis of the homing accuracy measured by the normalized average miss distance to pursuer-evader maneuver ratio μ lead to some new observations. A maneuver ratio of μ=2.0, which is certainly unsatisfactory for interception a maneuvering target for PN with N'=3, doesn't seem to hurt too much the accuracy of the "pure guidance strategies" in the environment for which they were optimized (strategy "J" against periodical maneuvering combined with EJ and strategy "L", as well as APN, against constant maneuvers without jinking).

Moreover, increasing the maneuver ratio beyond μ=3 doesn't improve substantially the guidance accuracy. These results confirm that, assuming a satisfactory estimation of the relative state, the guidance law based on linear differential game theory with perfect information [6] has no need for a high maneuver ratio.

The results indicate that the operational effectiveness of an optimal mixed strategy guidance law, measured by SSKP is quite insensitive to the pursuer-evader maneuver ratio for $5 \geq \mu \geq 3$. Its value is mainly determined by the warhead lethality. Increased maneuverability makes some difference only for very large

warheads. Similarly, the optimal mixed strategy itself (i.e. the probability distribution over the given pure strategy set) is essentially determined by the warhead lethality. In the present case increased warhead lethality favors strategy "L" at the expense of "J", which has very large miss distances against constant target maneuver without ECM. These conclusions lead to a cost-effective missile design with robust guidance performance with respect to all target evasive strategies which are taken into account in the MSG design process.

In summary, the non-dimensional sensitivity analysis of Ref. 2 gave a generalized demonstration for the superior performance of the mixed strategy (MSG) approach over PN and APN.

III. VARIABLE SPEED ASSESSMENT

3.1 AERODYNAMIC MODEL

The constant speed missile models used in most linearized guidance studies completely neglect the aerodynamic and propulsion characteristics of the missile. In the present study it is assumed that at the terminal guidance phase considered for the analysis the rocket motor of the missile is burned out. Consequently, the kinetic energy and the speed of the missile (V_p) are continuously dissipated by the aerodynamic drag, while the mass (m_p) remains constant.

Assuming more or less horizontal flight the speed loss of the missile is governed by the differential equation

$$\dot{V}_p = - D_p/m_p \tag{1}$$

The drag force D_p is given by

$$D_p = \frac{1}{2} \rho(h) V_p^2 S_p (C_D)_p \tag{2}$$

where $\rho(h)$ is the air density at the flight altitude S_p is the surface of

reference and $(C_D)_p$ is the nondimensional drag coefficient. The aerodynamic lift force is also expressed by a similar formula

$$L_p = \frac{1}{2}\rho(h)V_p^2 S_p C_{L_p}(\alpha_p) \tag{3}$$

where $C_{L_p}(\alpha_p)$ is the nondimensional lift coefficient depending on the angle of attack α_p. The drag coefficient is composed of two parts

$$(C_D)_p = \left(C_{D_o}\right)_p + C_{L_p}(\alpha_p)\sin\alpha_p \tag{4}$$

the first part is the zero-lift drag coefficient depending on the altitude while the second part represents the lift dependent induced drag. All coefficients are functions of the Mach number. The aerodynamic data used in the present phase of the investigation was provided to us by WL/MNAG. It is summarized in Table 1.

	$\left(C_{D_o}\right)_p$		$C_{L_p}(\alpha_p)$		
Mach No.	h=0	h=70 kft	$\alpha=5^\circ$	$\alpha=10^\circ$	$\alpha=15^\circ$
0.60	0.4765	0.5675	2.89	6.42	10.93
0.90	0.4300	0.5100	3.07	6.94	11.61
1.20	0.6917	0.8266	3.42	7.84	13.41
1.50	0.6250	0.7455	3.52	7.84	13.51
2.00	0.5306	0.6344	3.19	7.28	12.72
2.50	0.4568	0.5435	2.90	6.65	11.75
3.50	0.3293	0.3960	2.47	5.78	10.15
4.50	0.[illegible]590	0.3105	2.17	5.09	8.76

Table 1. Aerodynamic coefficients.

The lateral acceleration of the missile is given by

$$a_p = L_p/m_p = \rho(h)V_p^2 S_p C_{L_p}(\alpha_p)/2m_p \tag{5}$$

It is assumed that in both guidance channels of the missile the trimmed angle of attack is limited, due to physical limits of the control deflection by

$$|(\alpha_y)_{max}| = |(\alpha_z)_{max}| \le 10^\circ \tag{6}$$

As a consequence of these constraints the resultant angle of attack of the missile is limited by

$$|\alpha_p| \le 10^\circ\sqrt{2} = 14.14^\circ \tag{7}$$

The actual resultant angle of attack α_p is computed from the actual lateral acceleration (5), via

$$C_{L_p}(\alpha_p) = 2m_p a_p/\rho(h)V_p^2 S_p \tag{8}$$

by a search for the correct value of α_p interpolating the data of Table 1.

The actual lateral acceleration is the resultant of the accelerations on the perpendicular pitch and yaw channels

$$a_p = [a_{py}^2 + a_{pz}^2]^{1/2} \tag{9}$$

where the actual lateral acceleration in each channel is obtained from the acceleration command (a_{req}) derived by the guidance computer, via the first order dynamics transfer function of the missile autopilot

$$\tau_p \dot{a}_{pi} = (a_{req})_i - a_{pi} \qquad i = y, z \tag{10}$$

3.2 MODIFIED LATERAL ACCELERATION COMMAND LIMIT

In the previous phases of the investigation the limit was imposed on the resultant lateral acceleration command as a circular constraint

$$[(a_{req}^2)_y + (a_{req}^2)_z]^{1/2} \le (a_p)_{max} \tag{11}$$

In context of a variable speed missile model it seemed more appropriate to impose the constraint on each guidance channel separately, similarly to the constraint of the angle of attack

$$|(a_{req})_y| \le (a_p)_{max} \qquad |(a_{req})_z| \le (a_p)_{max} \tag{12}$$

This modification implicitly increased the missile maneuverability (in

particular at the terminal phase) by $\sqrt{2}=1.414$. In order to be able to compare the homing performance of constant speed and variable speed missile models, first the effect of modifying the acceleration constraint had to be evaluated using a constant speed model only.

The list of fixed parameters used in the simulation is given in Table 2. For the constant speed model $V_p=600$ m/sec (used in earlier phases) was selected. The results of the comparison for both pure guidance strategies "L" and "J" and for the optimal mixed strategy "M*" are given in Tables 3-5 respectively.

Note that NJ indicates an ECM free environment.

Table 2. Fixed engagement parameters.

<u>Initial conditions</u> (head-on)		
	Initial pursuer heading	$\psi_{po} = 0^\circ$
	Initial evader heading	$\psi_{eo} = 180^\circ$
	Nominal time of flight	$t_f = 5$ sec
	Flight altitude	$h = 8$ km
<u>Evader parameters</u>		
	Velocity	$V_e = 300$ m/sec
	Lateral acceleration limit	$(a_e)_{max} = 50$ m/sec
	Roll dynamics	$t_R = 2$ sec
	Glint parameters:	
	Band width	$BW = 2$ Hz
	Standard deviations	$\sigma_x = 3.7$ m
	(in body axes)	$\sigma_y = 2.5$ m
		$\sigma_z = 0.05$ m
	Amplitude of Electronic Jinking	$w_{max} = 4.7$ m
<u>Pursuer parameters</u>		
	Mass	$m_p = 103$ kg
	Surface of reference	$S_p = 0.285$ m^2
	Lateral acceleration limit	$(a_p)_{max} = 150$ m/sec
	Autopilot time constant	$\tau_p = 0.2$ sec
	Constant roll angle	$\phi_p = 0^\circ$
	Warhead lethality range	$R_e = 4.0$m
	Reliability factor	$(P_K)_{max} = 0.9$

Table 3. Effects of modifying the acceleration constraint. V_{p_o} =600 m/s (constant). Strategy L

		Circular		Modified	
ω_j [r/s]	ω_e [r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	3.80(2.22)	0.727	3.62(2.00)	0.757
	0.50	4.30(2.35)	0.674	3.68(2.09)	0.747
	0.75	5.11(2.57)	0.542	4.41(2.23)	0.638
	1.00	6.60(2.88)	0.376	5.73(2.59)	0.492
	1.50	5.10(2.55)	0.553	4.49(2.31)	0.644
	2.00	3.20(1.86)	0.798	3.02(1.74)	0.817
	2.50	2.80(1.75)	0.837	2.68(1.58)	0.848
	3.00	2.60(1.69)	0.842	2.51(1.58)	0.854
0.0	0.00	4.60(0.00)	0.813	4.64(0.03)	0.812
	0.50	5.80(2.19)	0.512	5.53(1.72)	0.556
	0.75	6.63(2.71)	0.381	6.16(2.32)	0.444
	1.00	7.50(4.41)	0.361	6.95(3.87)	0.416
	1.50	7.10(3.23)	0.367	6.71(2.90)	0.398
	2.00	6.60(2.01)	0.301	6.45(1.83)	0.325
	2.50	6.60(2.33)	0.237	6.56(1.36)	0.290
	3.00	6.40(1.14)	0.299	6.20(1.01)	0.340
1.0	0.00	5.30(3.02)	0.647	5.10(2.72)	0.665
	0.50	7.20(4.13)	0.412	6.55(3.39)	0.446
	0.75	8.49(4.46)	0.290	7.55(3.64)	0.330
	1.00	9.60(4.95)	0.233	8.64(4.27)	0.261
	1.50	8.60(4.04)	0.248	7.92(3.62)	0.279
	2.00	7.00(2.96)	0.328	6.72(2.52)	0.337
	2.50	6.50(2.60)	0.376	6.29(2.39)	0.406
	3.00	6.40(2.30)	0.399	6.12(2.29)	0.427
2.0	0.00	8.90(5.38)	0.331	8.09(4.95)	0.370
	0.50	9.90(5.02)	0.209	8.65(4.24)	0.250
	0.75	10.81(4.96)	0.155	9.36(4.12)	0.184
	1.00	11.60(5.64)	0.145	10.31(4.75)	0.171
	1.50	10.10(5.29)	0.198	9.35(4.75)	0.209
	2.00	9.00(4.12)	0.241	8.51(3.90)	0.252
	2.50	8.70(4.51)	0.276	8.27(4.03)	0.298
	3.00	8.60(4.49)	0.314	8.16(4.10)	0.326
3.0	0.00	9.70(4.27)	0.217	8.99(4.20)	0.251
	0.50	10.90(4.78)	0.156	9.52(4.13)	0.209
	0.75	11.70(4.93)	0.134	10.20(4.28)	0.178
	1.00	12.10(5.02)	0.137	11.04(4.66)	0.148
	1.50	10.60(5.18)	0.188	10.11(4.67)	0.181
	2.00	9.30(4.43)	0.225	8.94(3.91)	0.209
	2.50	9.20(4.41)	0.238	8.81(3.92)	0.239
	3.00	9.20(4.41)	0.238	8.69(4.01)	0.260

$\bar{M}$ = average miss distance
σ = standard deviation

Table 4. Effects of modifying the acceleration constraint. V_{p_o} =600 m/s (constant). Strategy J.

		Circular		Modified	
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	25.40(3.91)	0.000	20.71(4.30)	0.000
	0.50	13.60(7.84)	0.163	10.10(6.47)	0.300
	0.75	8.78(3.94)	0.229	5.81(3.21)	0.484
	1.00	4.60(1.37)	0.589	3.91(2.19)	0.726
	1.50	3.20(1.86)	0.801	3.21(1.70)	0.815
	2.00	2.80(1.75)	0.823	3.17(1.75)	0.804
	2.50	3.00(1.60)	0.828	3.21(1.80)	0.795
	3.00	2.90(1.58)	0.830	3.15(1.89)	0.795
0.0	0.00	4.60(0.00)	0.815	4.64(0.03)	0.814
	0.50	4.80(0.99)	0.694	4.93(1.25)	0.643
	0.75	4.92(1.22)	0.641	5.09(1.30)	0.587
	1.00	4.70(1.39)	0.674	4.87(1.44)	0.648
	1.50	4.00(0.90)	0.810	3.97(1.13)	0.815
	2.00	4.20(0.92)	0.808	4.27(0.96)	0.792
	2.50	4.50(0.95)	0.780	4.61(0.78)	0.757
	3.00	4.80(0.00)	0.750	4.90(0.58)	0.709
1.0	0.00	4.40(0.94)	0.809	4.10(1.10)	0.821
	0.50	4.50(1.36)	0.706	4.49(1.43)	0.695
	0.75	4.46(1.37)	0.701	4.63(1.39)	0.666
	1.00	4.30(1.33)	0.725	4.56(1.45)	0.684
	1.50	3.90(1.26)	0.808	3.96(1.26)	0.798
	2.00	4.00(1.28)	0.811	4.14(0.59)	0.805
	2.50	4.10(1.30)	0.804	4.24(1.10)	0.793
	3.00	4.30(0.93)	0.788	4.44(1.00)	0.763
2.0	0.00	3.80(0.88)	0.864	3.30(1.14)	0.885
	0.50	4.30(1.63)	0.720	4.15(1.42)	0.745
	0.75	4.17(1.43)	0.735	4.36(1.35)	0.714
	1.00	4.00(1.28)	0.762	4.34(1.41)	0.712
	1.50	3.60(1.50)	0.825	3.76(1.27)	0.818
	2.00	3.60(1.22)	0.855	3.72(1.19)	0.847
	2.50	3.60(1.22)	0.866	3.76(1.04)	0.856
	3.00	3.60(0.85)	0.869	3.75(0.98)	0.858
3.0	0.00	3.60(1.50)	0.820	2.86(1.17)	0.877
	0.50	4.20(1.88)	0.706	3.85(1.49)	0.765
	0.54	3.86(1.51)	0.770	4.08(1.25)	0.771
	1.00	3.70(1.52)	0.800	4.17(1.36)	0.751
	1.50	3.70(1.52)	0.806	3.80(1.39)	0.802
	2.00	3.40(1.46)	0.860	3.63(1.14)	0.854
	2.50	3.40(1.18)	0.879	3.55(1.04)	0.871
	3.00	3.30(1.17)	0.887	3.50(1.00)	0.883

$\bar{M}$ = average miss distance
σ = standard deviation

Table 5. Effects of modifying the acceleration constraint. V_{p_o} =600 m/s (constant). Optimal mixed strategy.

		Circular	Modified
ω_j [r/s]	ω_e [r/s]	SSKP	SSKP
	0.00	0.402	0.415
	0.50	0.446	0.545
	0.75	0.402	0.568
NJ	1.00	0.471	0.598
	1.50	0.664	0.721
	2.00	0.809	0.811
	2.50	0.833	0.824
	3.00	0.837	0.827
	0.00	0.814	0.813
	0.50	0.593	0.595
	0.75	0.497	0.509
0.0	1.00	0.501	0.521
	1.50	0.565	0.566
	2.00	0.528	0.536
	2.50	0.480	0.501
	3.00	0.501	0.507
	0.00	0.719	0.735
	0.50	0.543	0.558
	0.75	0.474	0.482
1.0	1.00	0.453	0.452
	1.50	0.498	0.513
	2.00	0.544	0.548
	2.50	0.567	0.581
	3.00	0.573	0.579
	0.00	0.569	0.603
	0.50	0.437	0.473
	0.75	0.414	0.423
2.0	1.00	0.421	0.415
	1.50	0.478	0.484
	2.00	0.516	0.521
	2.50	0.540	0.550
	3.00	0.562	0.566
	0.00	0.487	0.534
	0.50	0.402	0.460
	0.75	0.418	0.446
3.0	1.00	0.433	0.420
	1.50	0.464	0.461
	2.00	0.509	0.500
	2.50	0.525	0.524
	3.00	0.528	0.541

The comparison lead to the conclusion that the modification of the acceleration constraint slightly increased the value of the guaranteed SSKP, slightly changed the optimal mixed strategy and also affected the optimal strategy set of the evader. The quantitative changes are summarized in Table 6.

Table 6. Effects of modifying the acceleration constraint.

	Circular Constraint	Modified
Guaranteed SSKP	0.402	0.415
Optimal Mixed Strategy (L/J)	0.553/0.447	0.548/0.452
Optimal Strategy Set of the Evader	NJ, ω_e=0.0 r/s NJ, ω_e=0.75 r/s ω_j=3.0, ω_e=0.5 r/s	NJ, ω_e=0.0 r/s ω_j=2.0, ω_e=1.0 r/s

In the sequel the modified performance level will serve as a basis of comparison for the variable speed missile model.

3.3 VARIABLE SPEED MISSILE - FIRST ASSESSMENT

For the variable speed assessment the initial conditions of the end-game engagement have to be carefully selected in order to allow a meaningful comparison with the constant speed model. In the examples for evaluating the effect of speed variations an altitude of 8 km was selected. It was assumed that altitude variations (since target maneuver is confined to the horizontal plane) are neglectable and therefore air density is constant.

Moreover, in order to keep both the nominal duration of the end game and the initial range unchanged the initial missile velocity was selected in such a way that the average speed is the same as of the constant speed model. For the example presented here (h=8 km and $\bar{V}_p$=600 m/sec, t_f=5 sec, R_o=4500 m) the initial missile velocity of V_{p_o} =645 m/sec was found to be appropriate.

The results of the comparison for the two pure strategies (L and J) and the optimal mixed strategy are presented in Tables 7-9 respectively.

Table 7. Comparison of variable and constant speed models. h=8 km. Strategy L.

		Constant speed, V_p=600 m/s		Variable speed, V_{p_c}=645 m/s	
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	3.62(2.00)	0.757	4.09(2.29)	0.679
	0.50	3.68(2.09)	0.747	3.92(2.28)	0.720
	0.75	4.41(2.23)	0.638	4.18(2.22)	0.678
	1.00	5.73(2.59)	0.492	5.67(2.70)	0.493
	1.50	4.49(2.31)	0.644	4.90(2.51)	0.581
	2.00	3.02(1.74)	0.817	3.16(1.87)	0.801
	2.50	2.68(1.58)	0.848	2.83(1.68)	0.836
	3.00	2.51(1.58)	0.854	2.61(1.71)	0.840
0.0	0.00	4.64(0.03)	0.812	4.66(0.05)	0.806
	0.50	5.53(1.72)	0.556	5.35(1.54)	0.582
	0.75	6.16(2.32)	0.444	5.90(2.17)	0.482
	1.00	6.95(3.87)	0.416	6.91(3.73)	0.390
	1.50	6.71(2.90)	0.398	7.05(3.06)	0.383
	2.00	6.45(1.83)	0.325	6.58(1.92)	0.305
	2.50	6.56(1.36)	0.290	6.82(1.40)	0.243
	3.00	6.20(1.01)	0.340	6.38(1.05)	0.303
1.0	0.00	5.10(2.72)	0.665	5.54(3.19)	0.631
	0.50	6.55(3.39)	0.446	6.70(3.59)	0.436
	0.75	7.55(3.64)	0.330	7.54(3.85)	0.347
	1.00	8.64(4.27)	0.261	8.70(4.46)	0.263
	1.50	7.92(3.62)	0.279	8.35(3.73)	0.257
	2.00	6.72(2.52)	0.337	6.96(2.79)	0.326
	2.50	6.29(2.39)	0.406	6.51(2.53)	0.375
	3.00	6.12(2.29)	0.427	6.43(2.38)	0.392
2.0	0.00	8.09(4.95)	0.370	9.09(5.30)	0.305
	0.50	8.65(4.24)	0.250	9.14(4.39)	0.213
	0.75	9.36(4.12)	0.184	9.59(4.25)	0.183
	1.00	10.31(4.75)	0.171	10.44(4.76)	0.160
	1.50	9.35(4.75)	0.209	9.51(4.90)	0.213
	2.00	8.51(3.90)	0.252	8.65(4.05)	0.254
	2.50	8.27(4.03)	0.298	8.46(4.18)	0.289
	3.00	8.16(4.10)	0.326	8.34(4.31)	0.337
3.0	0.00	8.99(4.20)	0.251	11.70(3.25)	0.062
	0.50	9.52(4.13)	0.209	10.43(4.07)	0.153
	0.75	10.20(4.28)	0.178	10.28(4.20)	0.159
	1.00	11.04(4.66)	0.148	10.74(4.68)	0.165
	1.50	10.11(4.67)	0.181	10.05(5.02)	0.205
	2.00	8.94(3.91)	0.209	9.39(4.43)	0.213
	2.50	8.81(3.92)	0.239	9.60(4.33)	0.201
	3.00	8.69(4.01)	0.260	9.80(4.34)	0.198

$\bar{M}$ = average miss distance
σ = standard deviation

Table 8. Comparison of variable and constant speed models. h=8 km. Strategy J.

		Constant speed, V_p=600 m/s		Variable speed, V_{p_o}=645 m/s	
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	20.71(4.30)	0.000	23.09(4.07)	0.000
	0.50	10.10(6.47)	0.300	12.69(7.12)	0.190
	0.75	5.81(3.21)	0.484	8.27(4.48)	0.293
	1.00	3.91(2.19)	0.726	5.32(2.78)	0.543
	1.50	3.21(1.70)	0.815	3.72(1.96)	0.727
	2.00	3.17(1.75)	0.804	3.28(1.92)	0.787
	2.50	3.21(1.80)	0.795	3.48(1.92)	0.773
	3.00	3.15(1.89)	0.795	3.26(1.79)	0.793
0.0	0.00	4.64(0.03)	0.814	4.63(0.03)	0.816
	0.50	4.93(1.25)	0.643	4.81(1.14)	0.691
	0.75	5.09(1.30)	0.587	4.97(1.22)	0.632
	1.00	4.87(1.44)	0.648	4.83(1.42)	0.659
	1.50	3.97(1.13)	0.815	4.11(1.31)	0.792
	2.00	4.27(0.96)	0.792	4.25(0.91)	0.799
	2.50	4.61(0.78)	0.757	4.53(0.73)	0.778
	3.00	4.90(0.58)	0.709	4.77(0.54)	0.748
1.0	0.00	4.10(1.10)	0.821	4.81(0.88)	0.717
	0.50	4.49(1.43)	0.695	4.65(1.40)	0.694
	0.75	4.63(1.39)	0.666	4.53(1.38)	0.688
	1.00	4.56(1.45)	0.684	4.45(1.39)	0.709
	1.50	3.96(1.26)	0.798	4.06(1.37)	0.787
	2.00	4.14(0.59)	0.805	4.12(1.16)	0.801
	2.50	4.24(1.10)	0.793	4.18(1.05)	0.804
	3.00	4.44(1.00)	0.763	4.38(0.90)	0.783
2.0	0.00	3.30(1.14)	0.885	4.78(1.38)	0.666
	0.50	4.15(1.42)	0.745	4.59(1.56)	0.688
	0.75	4.36(1.35)	0.714	4.31(1.34)	0.724
	1.00	4.34(1.41)	0.712	4.16(1.45)	0.738
	1.50	3.76(1.27)	0.818	3.78(1.46)	0.799
	2.00	3.72(1.19)	0.847	3.70(1.17)	0.846
	2.50	3.76(1.04)	0.856	3.76(1.00)	0.859
	3.00	3.75(0.98)	0.858	3.70(1.02)	0.864
3.0	0.00	2.86(1.17)	0.877	5.23(2.19)	0.485
	0.50	3.85(1.49)	0.765	4.69(2.07)	0.635
	0.75	4.08(1.25)	0.771	4.11(1.50)	0.737
	1.00	4.17(1.36)	0.751	3.97(1.30)	0.788
	1.50	3.80(1.39)	0.802	3.80(1.46)	0.789
	2.00	3.63(1.14)	0.854	3.64(1.16)	0.846
	2.50	3.55(1.04)	0.871	3.61(1.09)	0.859
	3.00	3.50(1.00)	0.883	3.62(1.02)	0.873

$\bar{M}$ = average miss distance
σ = standard deviation

Table 9. Comparison of variable and constant speed models. h=8 km. Optimal mixed strategy.

		V_p=600 m/s	Variable
ω_j[r/s]	ω_e[r/s]	SSKP	SSKP
NJ	0.00	0.415	0.299
	0.50	0.545	0.423
	0.75	0.568	0.462
	1.00	0.598	0.521
	1.50	0.721	0.663
	2.00	0.811	0.793
	2.50	0.824	0.801
	3.00	0.827	0.814
0.0	0.00	0.813	0.812
	0.50	0.595	0.643
	0.75	0.509	0.566
	1.00	0.521	0.541
	1.50	0.586	0.612
	2.00	0.536	0.582
	2.50	0.501	0.543
	3.00	0.507	0.552
1.0	0.00	0.735	0.679
	0.50	0.558	0.580
	0.75	0.482	0.538
	1.00	0.452	0.513
	1.50	0.513	0.554
	2.00	0.548	0.592
	2.50	0.581	0.615
	3.00	0.579	0.611
2.0	0.00	0.603	0.507
	0.50	0.473	0.479
	0.75	0.423	0.486
	1.00	0.415	0.484
	1.50	0.484	0.541
	2.00	0.521	0.586
	2.50	0.550	0.608
	3.00	0.566	0.632
3.0	0.00	0.534	0.299
	0.50	0.460	0.423
	0.75	0.446	0.483
	1.00	0.420	0.514
	1.50	0.461	0.532
	2.00	0.500	0.567
	2.50	0.524	0.569
	3.00	0.541	0.576

The general trend in the variable speed model is increased miss distances and consequently reduced SSKP almost everywhere and particularly against constant target maneuvers (ω_e=0). The outcome is a substantial degradation of the guaranteed SSKP, a large variation of the optimal mixed strategy and significant change in the evader's optimal strategy set, as summarized in Table 10.

Table 10. Comparison of variable and constant speed models.

	Constant speed	Variable speed
Guaranteed SSKP	0.415	0.299
Optimal Mixed Strategy L/J	0.548/0.452	0.440/0.560
Evader's Optimal Strategy Set	NJ, ω_e=0.0 r/s ω_j=2.0, ω_e=1.0 r/s	NJ, ω_e=0.0 r/s ω_j=3.0, ω_e=0.0 r/s

The reason for the performance degradation was revealed by a detailed analysis of the variable speed results. It was found that the geometry of an interception against constant evader maneuver is characterized by a rather important change in the angle between the line of sight and the missile velocity vector. In such geometries the longitudinal acceleration (or deceleration) of the missile induces a rotation of the line of sight not accounted for in the presently used guidance law, which was developed for a constant speed missile model [6].

3.4 GUIDANCE LAW IMPROVEMENT

In the currently used guidance law, as in all linearized studies, only accelerations perpendicular to the line-of-sight are accounted for. The missile acceleration vector, perpendicular to the line-of-sight in the reference coordinate system attached to the initial line of sight, is

$$\vec{a}_{pN} = \begin{pmatrix} 0 \\ a_{pNy} \\ a_{pNz} \end{pmatrix} = B^T \begin{pmatrix} a_{px} \\ a_{py} \\ a_{pz} \end{pmatrix} \qquad (13)$$

where a_{px}, a_{py}, a_{pz} are the components of the missile acceleration vector in a coordinate system aligned with its velocity vector, and B is the transformation matrix between the two coordinate systems

$$B = \begin{pmatrix} \cos\psi_p \cos\theta_p & \sin\psi_p \cos\theta_p & -\sin\theta_p \\ \cos\psi_p \sin\theta_p \sin\phi_p - \sin\psi_p \cos\phi_p & \sin\psi_p \sin\theta_p \sin\phi_p + \cos\psi_p \cos\phi_p & \cos\theta_p \sin\phi_p \\ \cos\psi_p \sin\theta_p \cos\phi_p + \sin\psi_p \sin\phi_p & \sin\psi_p \sin\theta_p \cos\phi_p - \cos\psi_p \sin\phi_p & \cos\theta_p \cos\phi_p \end{pmatrix} \tag{14}$$

where ψ_p, θ_p are the flight path angles depicted in Fig. 1 and ϕ_p is the pursuer's roll angle.

Since by definition

$$a_{px} = \dot{V}_p \tag{15}$$

in a constant speed model this term vanishes, and only a_{py} and a_{pz}, controlled by the respective acceleration commands $(a_{req})_y$, $(a_{req})_z$, are considered in the derivation of the guidance law. The nonvanishing longitudinal acceleration of a variable speed model (though it is affected by the missile maneuvers via the increased induced drag) is not controllable by the guidance law.

In order to avoid the nonrequired rotation of the line of sight induced by the longitudinal missile acceleration, its component perpendicular to the line of sight has to be eliminated by an appropriate guidance command. By rewriting (13) for the required acceleration vector

$$(\vec{a}_{PN})_{req} = \begin{pmatrix} 0 \\ a_{RNy} \\ a_{RNz} \end{pmatrix} = B^T \begin{pmatrix} \dot{V}_p \\ a_{Ry} \\ a_{Rz} \end{pmatrix} \tag{16}$$

one obtains, from the two non-zero equations, the required lateral acceleration components a_{Ry} and a_{Rz} as functions of the required acceleration components perpendicular to the line of sight (a_{RNy} and a_{RNz}) and the longitudinal acceleration $\dot{V}_p$. The correction terms (with respect to the constant speed case)

Table 11. Effect of guidance law improvement.
h=8 km. Strategy L.

		Variable speed, V_{p_o}=645 m/s				Constant speed V_p=600 m/s	
		Original		Improved			
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
	0.00	4.09(2.29)	0.679	3.73(2.09)	0.732	3.62(2.00)	0.757
	0.50	3.92(2.28)	0.720	3.66(2.08)	0.752	3.68(2.09)	0.747
	0.75	4.18(2.22)	0.678	4.34(2.21)	0.653	4.41(2.23)	0.638
NJ	1.00	5.67(2.70)	0.493	5.67(2.62)	0.500	5.73(2.59)	0.492
	1.50	4.90(2.51)	0.581	4.48(2.30)	0.646	4.49(2.31)	0.644
	2.00	3.16(1.87)	0.801	3.03(1.77)	0.815	3.02(1.74)	0.817
	2.50	2.83(1.68)	0.836	2.68(1.62)	0.851	2.68(1.58)	0.848
	3.00	2.61(1.71)	0.840	2.50(1.59)	0.854	2.51(1.58)	0.854
	0.00	4.66(0.05)	0.806	4.64(0.05)	0.811	4.64(0.03)	0.812
	0.50	5.35(1.54)	0.582	5.47(1.61)	0.560	5.53(1.72)	0.556
	0.75	5.90(2.17)	0.482	6.08(2.22)	0.450	6.16(2.32)	0.444
0.0	1.00	6.91(3.73)	0.390	6.88(3.85)	0.421	6.95(3.87)	0.416
	1.50	7.05(3.06)	0.383	6.69(2.91)	0.402	6.71(2.90)	0.398
	2.00	6.58(1.92)	0.305	6.44(1.81)	0.323	6.45(1.83)	0.325
	2.50	6.82(1.40)	0.243	6.55(1.35)	0.290	6.56(1.36)	0.290
	3.00	6.38(1.05)	0.303	6.19(1.02)	0.345	6.20(1.01)	0.340
	0.00	5.54(3.19)	0.631	5.09(2.68)	0.667	5.10(2.72)	0.665
	0.50	6.70(3.59)	0.436	6.48(3.29)	0.446	6.55(3.39)	0.446
	0.75	7.54(3.85)	0.347	7.42(3.59)	0.337	7.55(3.64)	0.330
1.0	1.00	8.70(4.46)	0.263	8.56(4.25)	0.265	8.64(4.27)	0.261
	1.50	8.35(3.73)	0.257	7.95(3.57)	0.272	7.92(3.62)	0.279
	2.00	6.96(2.79)	0.326	6.69(2.53)	0.348	6.72(2.52)	0.337
	2.50	6.51(2.53)	0.375	6.29(2.37)	0.402	6.29(2.39)	0.406
	3.00	6.43(2.38)	0.392	6.10(2.27)	0.426	6.12(2.29)	0.427
	0.00	9.09(5.30)	0.305	8.12(4.99)	0.373	8.09(4.95)	0.370
	0.50	9.14(4.39)	0.213	8.59(4.17)	0.252	8.65(4.24)	0.250
	0.75	9.59(4.25)	0.183	9.24(4.06)	0.184	9.36(4.12)	0.184
2.0	1.00	10.44(4.76)	0.160	10.18(6.75)	0.174	10.31(4.75)	0.171
	1.50	9.51(4.90)	0.213	9.32(4.70)	0.210	9.35(4.75)	0.209
	2.00	8.65(4.05)	0.254	8.47(3.85)	0.253	8.51(3.90)	0.252
	2.50	8.46(4.18)	0.289	8.30(4.07)	0.294	8.27(4.03)	0.298
	3.00	8.34(4.31)	0.337	8.17(4.09)	0.333	8.16(4.10)	0.326
	0.00	11.70(3.25)	0.062	9.02(4.23)	0.253	8.99(4.20)	0.251
	0.50	10.43(4.07)	0.153	9.35(4.21)	0.223	9.52(4.13)	0.209
	0.75	10.28(4.20)	0.159	10.08(4.20)	0.178	10.20(4.28)	0.178
3.0	1.00	10.74(4.68)	0.165	10.94(4.60)	0.148	11.04(4.66)	0.148
	1.50	10.05(5.02)	0.205	10.06(4.62)	0.179	10.11(4.67)	0.181
	2.00	9.39(4.43)	0.213	8.91(3.94)	0.212	8.94(3.91)	0.209
	2.50	9.60(4.33)	0.201	8.81(3.86)	0.240	8.81(3.92)	0.239
	3.00	9.80(4.34)	0.198	8.64(3.99)	0.260	8.69(4.01)	0.260

$\bar{M}$ = average miss distance
σ = standard deviation

Table 12. Effect of guidance law improvement.
h=8 km. Strategy J.

		Variable speed, V_{p_o} =645 m/s				Constant speed	
		Original		Improved		V_p=600 m/s	
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	23.09(4.07)	0.000	23.16(4.48)	0.000	20.71(4.30)	0.000
	0.50	12.69(7.12)	0.190	10.68(7.05)	0.300	10.10(6.47)	0.300
	0.75	8.27(4.48)	0.293	5.92(3.35)	0.478	5.81(3.21)	0.484
	1.00	5.32(2.78)	0.543	3.94(2.20)	0.726	3.91(2.19)	0.726
	1.50	3.72(1.96)	0.727	3.13(1.68)	0.821	3.21(1.70)	0.815
	2.00	3.28(1.92)	0.787	3.14(1.73)	0.806	3.17(1.75)	0.804
	2.50	3.48(1.92)	0.773	3.16(1.82)	0.796	3.21(1.80)	0.795
	3.00	3.26(1.79)	0.793	3.09(1.84)	0.805	3.15(1.89)	0.795
0.0	0.00	4.63(0.03)	0.816	4.65(0.03)	0.811	4.64(0.03)	0.814
	0.50	4.81(1.14)	0.691	4.95(1.25)	0.636	4.93(1.25)	0.643
	0.75	4.97(1.22)	0.632	5.11(1.30)	0.581	5.09(1.30)	0.587
	1.00	4.83(1.42)	0.659	4.90(1.45)	0.641	4.87(1.44)	0.648
	1.50	4.11(1.31)	0.792	3.96(1.12)	0.816	3.97(1.13)	0.815
	2.00	4.25(0.91)	0.799	4.25(0.96)	0.795	4.27(0.96)	0.792
	2.50	4.53(0.73)	0.778	4.59(0.78)	0.762	4.61(0.78)	0.757
	3.00	4.77(0.54)	0.748	4.87(0.57)	0.717	4.90(0.58)	0.709
0.0	0.00	4.81(0.88)	0.717	4.17(1.07)	0.818	4.10(1.10)	0.821
	0.50	4.65(1.40)	0.694	4.52(1.45)	0.690	4.49(1.43)	0.695
	0.75	4.53(1.38)	0.688	4.64(1.39)	0.663	4.63(1.39)	0.666
	1.00	4.45(1.39)	0.709	4.53(1.49)	0.683	4.56(1.45)	0.684
	1.50	4.06(1.37)	0.787	3.98(1.25)	0.799	3.96(1.26)	0.798
	2.00	4.12(1.16)	0.801	4.09(1.11)	0.808	4.14(0.59)	0.805
	2.50	4.18(1.05)	0.804	4.27(1.03)	0.793	4.24(1.10)	0.793
	3.00	4.38(0.90)	0.783	4.41(0.95)	0.769	4.44(1.00)	0.763
1.0	0.00	4.78(1.38)	0.666	3.40(1.13)	0.881	3.30(1.14)	0.885
	0.50	4.59(1.56)	0.688	4.23(1.46)	0.727	4.15(1.42)	0.745
	0.75	4.31(1.34)	0.724	4.36(1.36)	0.712	4.36(1.35)	0.714
	1.00	4.16(1.45)	0.738	4.37(1.40)	0.710	4.34(1.41)	0.712
	1.50	3.78(1.46)	0.799	3.70(1.29)	0.822	3.76(1.27)	0.818
	2.00	3.70(1.17)	0.846	3.73(1.11)	0.850	3.72(1.19)	0.847
	2.50	3.76(1.00)	0.859	3.76(1.01)	0.855	3.76(1.04)	0.856
	3.00	3.70(1.02)	0.864	3.71(1.07)	0.857	3.75(0.98)	0.858
2.0.	0.00	5.23(2.19)	0.485	2.99(1.24)	0.865	2.86(1.17)	0.877
	0.50	4.69(2.07)	0.635	3.95(1.54)	0.749	3.85(1.49)	0.765
	0.75	4.11(1.50)	0.737	4.09(1.26)	0.767	4.08(1.25)	0.771
	1.00	3.97(1.30)	0.788	4.18(1.34)	0.752	4.17(1.36)	0.751
	1.50	3.80(1.46)	0.789	3.80(1.35)	0.805	3.80(1.39)	0.802
	2.00	3.64(1.16)	0.846	3.61(1.12)	0.859	3.63(1.14)	0.854
	2.50	3.61(1.09)	0.859	3.54(1.09)	0.874	3.55(1.04)	0.871
	3.00	3.62(1.02)	0.873	3.54(0.98)	0.883	3.50(1.00)	0.883

$\bar{M}$ = average miss distance
σ = standard deviation

Table 13. Effect of guidance law improvement. h=8 km. Optimal mixed strategy

		Variable Speed V_p=645 m/s		Constant
		Original	Improved	Speed
ω_j[r/s]	ω_e[r/s]	SSKP	SSKP	SSKP
NJ	0.00	0.299	0.410	0.415
	0.50	0.423	0.553	0.545
	0.75	0.462	0.576	0.568
	1.00	0.521	0.599	0.598
	1.50	0.663	0.723	0.721
	2.00	0.793	0.811	0.811
	2.50	0.801	0.827	0.824
	3.00	0.814	0.832	0.827
0.0	0.00	0.812	0.811	0.813
	0.50	0.643	0.593	0.595
	0.75	0.566	0.508	0.509
	1.00	0.541	0.518	0.521
	1.50	0.612	0.584	0.586
	2.00	0.582	0.531	0.536
	2.50	0.543	0.498	0.501
	3.00	0.552	0.509	0.507
1.0	0.00	0.679	0.733	0.735
	0.50	0.580	0.553	0.558
	0.75	0.538	0.480	0.482
	1.00	0.513	0.449	0.452
	1.50	0.554	0.504	0.513
	2.00	0.592	0.550	0.548
	2.50	0.615	0.574	0.581
	3.00	0.611	0.577	0.579
2.0	0.00	0.507	0.597	0.603
	0.50	0.479	0.461	0.473
	0.75	0.486	0.416	0.423
	1.00	0.484	0.410	0.415
	1.50	0.541	0.479	0.484
	2.00	0.586	0.516	0.521
	2.50	0.608	0.541	0.550
	3.00	0.632	0.564	0.566
3.0	0.00	0.299	0.522	0.534
	0.50	0.423	0.454	0.460
	0.75	0.483	0.437	0.446
	1.00	0.514	0.414	0.420
	1.50	0.532	0.454	0.461
	2.00	0.567	0.497	0.500
	2.50	0.569	0.519	0.524
	3.00	0.576	0.534	0.541

are proportional to $\dot{V}_p$ and to the tangent of the angle between the line of sight and the missile velocity vector.

A similar correction was proposed in the past for improving the performance of variable speed missile guided by proportional navigation [8]. Such a correction can be easily implemented in the missile by measuring the longitudinal acceleration and the off-bore sight angle of the seeker.

By introducing the correction terms compensating for the longitudinal acceleration into the perfect information game optimal guidance law of [6], an improved missile performance was obtained as it can be seen from the results presented in Tables 11-13 and summarized in Table 14.

Table 14. Effect of guidance law improvement.
(Variable speed model)

	Original Guidance Law	Improved Guidance Law
Guaranteed SSKP	0.299	0.410
Optimal Mixed Strategy (L/J)	0.44/0.56	0.56/0.44
Evader's Optimal Strategy Set	NJ, ω_e=0.0 r/s ω_j=3.0, ω_e=0.0 r/s	NJ, ω_e=0.0 r/s ω_j=2.0, ω_e=1.0 r/s

Comparison to Table 10 indicates that the difference between the per- formance of a variable speed missile using the improved version of the [illegible] law and of the equivalent constant speed model is insignificant. The [illegible] difference is most probably due to the constant closing speed approxi[illegible]ed for calculating the "time-to-go" in the guidance law.

3.5 EFFECT OF TERMINAL MANEUVERABILITY

In the variable speed example discussed in the previous subsection the final missile velocity $V_p(t_f)$ was sufficiently high in order to avo[illegible] of attack saturation of the missile. In other words the lateral [illegible]ration obtained by 10° of angle of attack was higher than the limit impo[illegible] the required acceleration. The effect of reduced terminal maneuverability [illegible]ted

Table 15. Effect of terminal maneuverability.
Variable speed model. h=8 km.
Strategy L.

		V_{p_o} =645 m/s		V_{p_o} =585 m/s		V_{p_o} =520 m/s	
ω_j[r/s]	ω_e[r/s]	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP	$\bar{M}$[m] σ[m]	SSKP
NJ	0.00	3.73(2.09)	0.732	3.90(2.25)	0.715	4.91(2.56)	0.591
	0.50	3.66(2.08)	0.752	4.12(2.32)	0.674	4.52(2.46)	0.626
	0.75	4.34(2.21)	0.653	4.57(2.31)	0.640	5.40(2.62)	0.500
	1.00	5.67(2.62)	0.500	5.53(2.61)	0.499	6.45(3.12)	0.414
	1.50	4.48(2.30)	0.646	4.47(2.28)	0.639	4.79(2.50)	0.598
	2.00	3.03(1.77)	0.815	3.26(1.81)	0.794	3.10(1.79)	0.810
	2.50	2.68(1.62)	0.851	2.70(1.71)	0.832	2.74(1.63)	0.840
	3.00	2.50(1.59)	0.854	2.51(1.66)	0.845	2.60(1.72)	0.842
0.0	0.00	4.64(0.05)	0.811	4.66(0.04)	0.807	4.64(0.03)	0.813
	0.50	5.47(1.61)	0.560	5.46(1.49)	0.560	6.15(2.31)	0.464
	0.75	6.08(2.22)	0.450	6.13(2.28)	0.444	6.99(2.84)	0.335
	1.00	6.88(3.85)	0.421	7.03(3.97)	0.410	7.28(4.20)	0.392
	1.50	6.69(2.91)	0.402	6.70(2.88)	0.403	6.88(3.02)	0.390
	2.00	6.44(1.81)	0.323	6.45(1.81)	0.321	6.66(1.79)	0.299
	2.50	6.55(1.35)	0.290	6.59(1.35)	0.283	6.76(1.38)	0.257
	3.00	6.19(1.02)	0.345	6.22(1.02)	0.337	6.34(1.06)	0.311
1.0	0.00	5.09(2.68)	0.667	5.07(2.82)	0.673	5.14(3.10)	0.676
	0.50	6.48(3.29)	0.446	6.54(3.32)	0.443	7.42(4.05)	0.382
	0.75	7.42(3.59)	0.337	7.54(3.69)	0.340	8.65(4.22)	0.276
	1.00	8.56(4.25)	0.265	8.73(4.48)	0.269	9.38(4.76)	0.241
	1.50	7.95(3.57)	0.272	7.95(3.60)	0.277	8.31(3.79)	0.253
	2.00	6.69(2.53)	0.348	6.78(2.56)	0.332	6.60(3.43)	0.332
	2.50	6.29(2.37)	0.402	6.28(2.39)	0.409	6.39(2.45)	[illegible]
	3.00	6.10(2.27)	0.426	6.10(2.35)	0.431	6.27(2.40)	[illegible]
2.0	0.00	8.12(4.99)	0.373	8.44(5.27)	0.362	9.04(5.78)	0.336
	0.50	8.59(4.17)	0.252	8.71(4.22)	0.239	9.88(4.89)	0.212
	0.75	9.24(4.06)	0.184	9.45(4.14)	0.196	10.71(4.78)	0.167
	1.00	10.18(6.75)	0.174	10.47(4.85)	0.174	11.34(5.05)	0.140
	1.50	9.32(4.70)	0.210	9.44(4.78)	0.213	10.07(5.22)	0.194
	2.00	8.47(3.85)	0.253	8.50(3.96)	0.252	8.83(4.23)	0.236
	2.50	8.30(4.07)	0.294	8.34(4.04)	0.303	8.63(4.33)	0.288
	3.00	8.17(4.09)	0.333	8.18(4.11)	0.328	8.47(4.35)	0.319
3.0	0.00	9.02(4.23)	0.253	9.11(4.45)	0.257	9.57(4.50)	0.220
	0.50	9.35(4.21)	0.223	9.38(4.25)	0.219	10.65(4.95)	0.180
	0.75	10.08(4.20)	0.178	10.27(4.31)	0.173	11.59(5.11)	0.151
	1.00	10.94(4.60)	0.148	11.19(4.80)	0.152	11.87(5.34)	[illegible]
	1.50	10.06(4.62)	0.179	10.14(4.76)	0.187	10.71(5.13)	[illegible]
	2.00	8.91(3.94)	0.212	8.95(3.93)	0.211	9.15(4.09)	[illegible]
	2.50	8.81(3.86)	0.240	8.88(3.95)	0.232	8.89(4.06)	[illegible]
	3.00	8.64(3.99)	0.260	8.77(4.01)	0.247	8.73(4.14)	[illegible]

$\bar{M}$ = average miss distance
σ = standard deviation

Table 16. Effect of terminal maneuverability.
Variable speed model: h=8 km.
Strategy J.

		V_{p_o} =645 m/s		V_{p_o} =585 m/s		V_{p_o} =520 m/s	
ω_j[r/s]	ω_e[r/s]	$\tilde{M}$[m] σ[m]	SSKP	$\tilde{M}$[m] σ[m]	SSKP	$\tilde{M}$[m] σ[m]	SSKP
NJ	0.00	23.16(4.48)	0.000	28.22(5.16)	0.000	37.26(5.20)	0.000
	0.50	10.68(7.05)	0.300	11.74(7.82)	0.258	15.13(10.3)	0.215
	0.75	5.92(3.35)	0.478	5.64(3.31)	0.516	7.61(3.84)	0.328
	1.00	3.94(2.20)	0.726	4.00(2.32)	0.693	4.28(2.34)	0.676
	1.50	3.13(1.68)	0.821	3.30(1.83)	0.786	3.36(1.78)	0.792
	2.00	3.14(1.73)	0.806	3.14(1.83)	0.800	3.30(1.75)	0.793
	2.50	3.16(1.82)	0.796	3.28(1.96)	0.783	3.11(1.79)	0.805
	3.00	3.09(1.84)	0.805	3.20(1.94)	0.785	3.22(1.82)	0.787
0.0	0.00	4.65(0.03)	0.811	4.68(0.06)	0.800	4.72(0.12)	0.789
	0.50	4.95(1.25)	0.636	4.98(1.21)	0.635	4.83(1.16)	0.671
	0.75	5.11(1.30)	0.581	5.13(1.28)	0.582	4.97(1.21)	0.627
	1.00	4.90(1.45)	0.641	4.84(1.40)	0.652	4.76(1.33)	0.669
	1.50	3.96(1.12)	0.816	3.97(1.09)	0.817	3.90(1.07)	0.823
	2.00	4.25(0.96)	0.795	4.28(0.97)	0.791	4.23(0.94)	0.800
	2.50	4.59(0.78)	0.762	4.59(0.76)	0.762	4.54(0.78)	0.772
	3.00	4.87(0.57)	0.717	4.86(0.58)	0.720	4.79(0.60)	0.737
1.0	0.00	4.17(1.07)	0.818	4.25(0.94)	0.815	4.93(1.44)	0.652
	0.50	4.52(1.45)	0.690	4.64(1.43)	0.674	4.65(1.59)	0.686
	0.75	4.64(1.39)	0.663	4.71(1.31)	0.659	4.53(1.32)	0.693
	1.00	4.53(1.49)	0.683	4.55(1.44)	0.691	4.44(1.37)	0.714
	1.50	3.98(1.25)	0.799	3.96(1.23)	0.806	3.89(1.19)	0.817
	2.00	4.09(1.11)	0.808	4.14(1.05)	0.807	4.07(1.13)	0.810
	2.50	4.27(1.03)	0.793	4.26(1.06)	0.792	4.22(1.02)	0.800
	3.00	4.41(0.95)	0.769	4.33(1.36)	0.762	4.34(1.08)	0.777
2.0	0.00	3.40(1.13)	0.881	3.51(1.06)	0.880	4.95(2.01)	0.589
	0.50	4.23(1.46)	0.727	4.32(1.54)	0.706	4.57(1.98)	0.683
	0.75	4.36(1.36)	0.712	4.36(1.29)	0.719	4.31(1.38)	0.721
	1.00	4.37(1.40)	0.710	4.32(1.37)	0.720	4.25(1.39)	0.732
	1.50	3.70(1.29)	0.822	3.76(1.24)	0.823	3.67(1.32)	0.824
	2.00	3.73(1.11)	0.850	3.72(1.13)	0.845	3.71(1.10)	0.852
	2.50	3.76(1.01)	0.855	3.72(1.14)	0.855	3.78(1.07)	0.848
	3.00	3.71(1.07)	0.857	3.72(1.07)	0.858	3.75(0.97)	0.861
3.0	0.00	2.99(1.24)	0.865	3.10(1.26)	0.863	5.78(1.76)	0.460
	0.50	3.95(1.54)	0.749	4.07(1.70)	0.722	4.80(2.24)	0.649
	0.75	4.09(1.26)	0.767	4.08(1.25)	0.768	4.11(1.41)	0.755
	1.00	4.18(1.34)	0.752	4.18(1.38)	0.742	4.16(1.34)	0.758
	1.50	3.80(1.35)	0.805	3.80(1.42)	0.800	3.78(1.42)	0.802
	2.00	3.61(1.12)	0.859	3.67(1.13)	0.855	3.65(1.15)	0.851
	2.50	3.54(1.09)	0.874	3.57(1.03)	0.871	3.51(1.10)	0.871
	3.00	3.54(0.98)	0.883	3.48(0.99)	0.882	3.50(0.95)	0.982

$\tilde{M}$ = average miss distance
σ = standard deviation

Table 17[illegible]

				V_{Po} =520 m/s
ω_j [r/s]	ω_e [r/s]	[illegible]	[illegible]	SSKP
NJ	[illegible]	[illegible]	[illegible]	0.331
	[illegible]	[illegible]	[illegible]	0.445
	[illegible]	[illegible]	[illegible]	0.424
	[illegible]	[illegible]	0.583	0.529
	[illegible]	[illegible]	0.702	0.683
	[illegible]	[illegible]	0.797	0.803
	[illegible]	[illegible]	0.811	0.825
	[illegible]	[illegible]	0.819	0.818
0.0	[illegible]	[illegible]	0.804	0.802
	[illegible]	[illegible]	0.592	0.555
	[illegible]	0.508	0.504	0.463
	[illegible]	0.518	0.514	0.514
	[illegible]	0.584	0.582	0.580
	[illegible]	0.531	0.524	0.519
	2.50	0.498	0.490	0.483
	[illegible]	0.509	0.502	0.498
1.0	0.00	0.733	0.734	0.665
	0.50	0.553	0.543	0.516
	0.75	0.480	0.478	0.459
	1.00	0.449	0.451	0.449
	1.50	0.504	0.505	0.501
	2.00	0.550	0.537	0.542
	2.50	0.574	0.574	0.567
	3.00	0.577	0.574	0.580
2.0	0.00	0.597	0.586	0.447
	0.50	0.461	0.441	0.419
	0.75	0.416	0.422	0.410
	1.00	0.410	0.410	0.400
	1.50	0.479	0.476	0.471
	2.00	0.516	0.508	0.507
	2.50	0.541	0.541	0.534
	3.00	0.564	0.557	0.557
3.0	0.00	0.522	0.518	0.331
	0.50	0.454	0.436	0.386
	0.75	0.437	0.430	0.416
	1.00	0.414	0.407	0.416
	1.50	0.454	0.452	0.451
	2.00	0.497	0.489	0.493
	2.50	0.519	0.508	0.522
	3.00	0.534	0.521	0.579

by two examples where the initial missile velocities are lower and consequently the maneuverability of the missile, imposed by the limit of the angle of attack, doesn't reach the level of the maximum acceleration command. In the example, presented in Tables 15-17 the initial missile velocities of V_{p_o}=585 m/sec and V_{p_o}=520 m/sec was used.

Given the same initial velocity for each target maneuver (ω_e) a different set of final conditions are obtain as is reflected in Table 18. The effect of terminal maneuverability on the missile performance is quite dramatic, but it is compatible with the results of the sensitivity analysis reported in Ref. 2.

Table 18. Effect of terminal maneuverability.
Variable speed model (h=8 km)

Initial velocity	645 m/s	585 m/s	520 m/s
Final velocity	520-560 m/s	460-510 m/s	405-460 m/s
Terminal maneuverability	150 m/s	125-145 m/s	95-125 m/s
Guaranteed SSKP	0.410	0.406	0.331
Optimal Mixed (L/J) Strategy	0.56/0.44	0.57/0.43	0.56/0.44
Evader's Optimal Strategy Set	NJ, ω_e=0.0 r/s ω_j=2.0, ω_e=1.0 r/s	NJ, ω_e=0.0 r/s ω_j=3.0, ω_e=1.0 r/s	NJ, ω_e=0.0 r/s ω_j=3.0, ω_e=0.0 r/s

IV. CONCLUSIONS

This report validates the results obtained in the previous years for a constant speed missile model, - which demonstrated the superior homing performance of the Mixed Strategy Guidance over other guidance laws, - were validated in a realistic variable speed end game scenario.

The main conclusion of the last investigation phase substantiated by a large set of simulations is that the performance of missiles using a Mixed Strategy Guidance is essentially the same in a variable speed scenario, as for a constant speed model, if two conditions are satisfied:

a. The guidance law of the missile includes a term compensating for non zero longitudinal accelerations.

b. The maximum maneuverability of the missile (determined by the limit imposed on the commanded lateral acceleration) can be attained during the entire engagement; or in other words, the limit imposed by aerodynamic control saturation is not reached.